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GENERAL MODEL OF SOMIGLIANI DISLOCATIONS

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1. Underlying the statistical description of plastic molding, substructure evolution, fracture,and other processes in real solids is the continual theory of defects (see [1-3], e.g.,). Among all the possible defects, dislocations and disclinations whose distributions can represent practically any substructures, occupy an important place. The examination of dislocations and disclinations as different defects is not always convenient and justified since they are Volterra dislocations (only of just a different kind). On the other hand, defects of the most general kind, Somigliani dislocations [2], can be the means for a single description of dislocations and disclinations. A step is made in this direction in [4] and a model is proposed for Somigliani dislocations, given by their basic plastic distortion fields β^P_{kl} and displacement velocities v^P_l. However, such Somigliani dislocations describes only the so-called dislocation model of defects [3]. This is completely adequate for a calculation of the dynamical elastic stress fields produced by defects, but certain disclination characteristics of the defect structure are not reflected here. The purpose of this paper is to obtain a general model of Somigliani defects which will equally take into account both the dislocation and the disclination characteristics of defects. As will be shown below, such a model should be a generalization of disclination (a rotational Volterra dislocation).

2. The usual (initial) definition of a Somigliani dislocation is formulated in terms of the total displacement fields u^T_l, which undergo arbitrarily changing jumps $[u^T_l]$ along S on the defect surface S [2]. In constructing the general model of a Somigliani dislocation, we proceed differently, namely, we give the definition of the model in terms of the basis plastic fields, as is done in [4].

We shall consider the general model of the Somigliani dislocation as a direct generalization of a disclination which is defined in the continual theory of defects by giving four basis plastic fields: ε^P_{kl} is the strain tensor, $\varkappa^P_{mq}$ is the bending–twisting tensor, v^P_l is the displacement velocity tensor, and w^P_q is the rotation velocity tensor [5, 6]. The expressions for the basis fields are obtained for an ordinary disclination by considering disclinations with a closed surface S(t) encloding a volume V(t) where t is the time. The starting point is the expression for the total displacements $u^T_l(\mathbf{r}, t)$ within the volume V(t) [5]

$$u_l^T(\mathbf{r}, t) = \int\limits_V \delta(\mathbf{R}) \left\{b_l + \varepsilon_{lqr}\Omega_q\left(x'_r - x^0_r\right)\right\} dV', \tag{2.1}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ is the difference between the radius-vectors of the observation and integration points, $\delta(\mathbf{r})$ is the three-dimensional Dirac delta function, b_l, Ω_q are the relative translation and rotation vectors of the edges of the slit S(t), ε_{lqr} is the unit antisymmetric-tensor, x_r are the Cartesian coordinates of the radius-vector $\mathbf{r}$, and x^0_r are coordinates of a point through which the axis of rotation passes. The basis fields are found by the following scheme [5, 6].

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The total distortion β^T_{kl} is calculated by differentiation with respect to the coordinates

$$\beta^T_{kl} = u^T_{l,k}, \tag{2.2}$$

where the subscript after the comma denotes the differentiation with respect to the appropriate Cartesian coordinate. The symmetric part of β^T_{kl} in (2.2) is the total strain

$$e^T_{kl} = \beta^T_{(kl)}, \tag{2.3}$$

where (kl) is the symmetrization operation in the subscripts indicated. The basis plastic field e^P_{kl} is defined as the singular part of e^T_{kl} in (2.3) concentrated on S(t).

Differentiating (2.1) with respect to the time t yields the velocity vector of the total displacement v^T_l

$$v^T_l = \frac{\partial}{\partial t} u^T_l. \tag{2.4}$$

The basis field of the plastic displacement v^P_i is again defined as the singular part of v^T_l in (2.4) concentrated on S(t).

The total rotation vector φ^T_q is found as the vector associated with the antisymmetric part of the total distortion β^T_{kl} in (2.2):

$$\varphi^T_q = \frac{1}{2}\varepsilon_{klq}\beta^T_{kl}. \tag{2.5}$$

By differentiating φ^T_q in (2.5) with respect to the coordinates, the total bending-twisting tensor $\varkappa^T_{mq}$ is evaluated

$$\varkappa^T_{mq} = \varphi^T_{q,m}. \tag{2.6}$$

The singular part of $\varkappa^T_{mq}$ concentrated on S(t) corresponds to the plastic bending-twisting tensor $\varkappa^P_{mq}$.

Finally, by differentiating φ^T_q in (2.5) with respect to time, the total rotation velocity is found

$$w^T_q = \frac{\partial}{\partial t}\varphi^T_q, \tag{2.7}$$

and the singular part of w^T_q concentration on S(t) is taken as the basis vector of the plastic rotation velocity w^P_q.

The procedure described above for the calculation of the basis fields (2.2)-(2.7) can be represented in a simpler form if the apparatus of motor calculus is used (six-dimensional space with covariant differentiation) [7]. The total basis fields e^T_{kl}, $\varkappa^T_{mq}$, v^T_l, w^T_q comprise a pair of motors (the motors are mapped by a column-matrices in writing), which are obtained by differentiating the motor from the vectors φ^T_q, u^T_l (we omit the subscripts on the tensors)

$$\begin{pmatrix} \varkappa^T \\ e^T \end{pmatrix} = \operatorname{grad}\begin{pmatrix} \boldsymbol{\varphi}^T \\ \mathbf{u}^T \end{pmatrix}; \tag{2.8}$$

$$\begin{pmatrix} \mathbf{w}^T \\ \mathbf{v}^T \end{pmatrix} = \frac{\partial}{\partial t}\begin{pmatrix} \boldsymbol{\varphi}^T \\ \mathbf{u}^T \end{pmatrix}. \tag{2.9}$$

Here grad is an operation of the form [7]

$$\operatorname{grad}\begin{pmatrix} \varphi^T_k \\ u^T_k \end{pmatrix} = \begin{pmatrix} \varphi^T_{k,i} \\ u^T_{k,i} - \varepsilon_{ik\alpha}\varphi^T_\alpha \end{pmatrix},$$

while $\partial/\partial t$ denotes

$$\frac{\partial}{\partial t}\begin{pmatrix} \varphi^T_k \\ u^T_k \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial t}\varphi^T_k \\ \frac{\partial}{\partial t} u^T_k \end{pmatrix}.$$

We follow the above-mentioned procedure in deriving the expressions for the basis fields in the general model of Somigliani dislocations, but we get rid of the assumption about constancy of the vectors b_l, Ω_q by considering them functions of the coordinates and time. This is a completely natural extension of an ordinary disclination. Since b_l, Ω_q are arbitrary functions, even the expression in the braces under the integral sign in (2.1) will be an arbitrary vector field $P_i(\mathbf{r}, t)$ so that we can write in place of (2.1)

$$u_l^T = \int\limits_{V(t)} \delta(\mathbf{R}) P_l(\mathbf{r}', t')\, dV'. \tag{2.10}$$

Starting from (2.10) and executing the necessary calculations, we obtain expressions for the basic plastic fields in the general model of the Somigliani dislocation:

$$e_{kl}^P = -\int\limits_{S(t)} \delta(\mathbf{R}) P_l(\mathbf{r}', t')\, dS'_{k(kl)}; \tag{2.11}$$

$$\varkappa_{mq}^P = -\frac{1}{2}\varepsilon_{klq}\int\limits_{S(t)} \delta_{,m}(\mathbf{R}) P_l(\mathbf{r}', t')\, dS'_k - \frac{1}{2}\varepsilon_{klq}\int\limits_{S(t)} \delta(\mathbf{R}) P_{l,k}(\mathbf{r}', t')\, dS'_m; \tag{2.12}$$

$$v_l^P = \int\limits_{S(t)} \delta(\mathbf{R}) P_l(\mathbf{r}', t')\, v_k(\mathbf{r}', t')\, dS'_k; \tag{2.13}$$

$$w_q^P = -\frac{1}{2}\varepsilon_{klq}\int\limits_{S(t)} \delta(\mathbf{R}) \dot{P}_l(\mathbf{r}', t')\, dS'_k + \frac{1}{2}\varepsilon_{klq}\int\limits_{S(t)} \delta_{,k}(\mathbf{R}) P_l v_j dS'_j - \frac{1}{2}\varepsilon_{klq}\varepsilon_{kmp}\int\limits_{L(t)} \delta(\mathbf{R}) P_l v_m dL'_p, \tag{2.14}$$

where v_k is the velocity of motion of the surface $S(t)$, the upper dot denotes differentiation with respect to the time, dL_p is an element of the contour L bounding $S(t)$. It can be seen that in the case when the general expression (2.10) goes over into the particular expression (2.1), the formulas (2.11)-(2.14) yield the correct formulas for the basis plastic fields of of an ordinary disclination [5].

3. As is seen from (2.8) and (2.9), the total basis fields are kinematically dependent and subject to compatibility conditions (integrability of the system of equations (2.8) and (2.9))

$$\operatorname{rot}\begin{pmatrix} \varkappa^T \\ e^T \end{pmatrix} = 0; \tag{3.1}$$

$$\operatorname{grad}\begin{pmatrix} \mathbf{w}^T \\ \mathbf{v}^T \end{pmatrix} = \frac{\partial}{\partial t}\begin{pmatrix} \varkappa^T \\ e^T \end{pmatrix}, \tag{3.2}$$

where rot(curl) is an operation of the form [7]

$$\operatorname{rot}\begin{pmatrix} \varkappa_{\beta k}^T \\ e_{\beta k}^T \end{pmatrix} = \begin{pmatrix} \varepsilon_{i\alpha\beta}\varkappa_{\beta k,\alpha}^T \\ \varepsilon_{i\alpha\beta}\left(e_{\beta k,\alpha}^T + \varepsilon_{k\alpha\gamma}\varkappa_{\beta\gamma}^T\right) \end{pmatrix}.$$

The plastic basis fields e_k^P, $\varkappa_{mq}^P$, v_l^P, w_q^P are not subject to the compatibility conditions (3.1) and (3.2) in the general case. The density α_{pl}, θ_{pq} and flux J_{kl}, S_{kq} tensors of the continuously distributed dislocations and disclinations corresponding to the dislocation-disclination models of the total Somigliani dislocations can be defined as a measure of the deviation from the compatibility conditions (3.1) and (3.2) for the basis plastic fields:

$$\begin{pmatrix} \theta \\ \alpha \end{pmatrix} = -\operatorname{rot}\begin{pmatrix} \varkappa^P \\ e^P \end{pmatrix}; \tag{3.3}$$

$$\begin{pmatrix} S \\ J \end{pmatrix} = \frac{\partial}{\partial t}\begin{pmatrix} \varkappa^P \\ e^P \end{pmatrix} - \operatorname{grad}\begin{pmatrix} \mathbf{w}^P \\ \mathbf{v}^P \end{pmatrix}. \tag{3.4}$$

Because of the defining relationships (3.3) and (3.4), the density α_{pl}, θ_{pq} and flux J_{kl}, S_{kq} tensors of the dislocations and disclinations here satisfy the compatibility conditions

$$\operatorname{div}\begin{pmatrix} \theta \\ \alpha \end{pmatrix} = 0; \tag{3.5}$$

$$\frac{\partial}{\partial t}\begin{pmatrix} \theta \\ \alpha \end{pmatrix} + \operatorname{rot}\begin{pmatrix} S \\ J \end{pmatrix} = 0, \tag{3.6}$$

where div is an operation of the form [7]

$$\operatorname{div}\begin{pmatrix}\theta_{mk}\\ \alpha_{mk}\end{pmatrix}=\begin{pmatrix}\theta_{mk,m}\\ \alpha_{mk,m}+\varepsilon_{hm\beta}\theta_{m\beta}\end{pmatrix}.$$

Substituting the expressions (2.11)-(2.14) obtained above for the basis plastic fields into (3.3) and (3.4), we find for the dislocation and disclination densities and fluxes in the general model of Somigliani dislocations:

$$\alpha_{pl}=\int_L \delta(\mathbf{R})\,P_l\,dL'_p+\varepsilon_{pmk}\int_S \delta(\mathbf{R})\,P_{l,m}dS'_k+\frac{1}{2}\varepsilon_{pmk}\int_S \delta(\mathbf{R})\,(P_{l,k}-P_{k,l})\,dS'_m; \tag{3.7}$$

$$\theta_{pq}=\frac{1}{2}\varepsilon_{slq}\int_L \delta(\mathbf{R})\,P_{l,s}\,dS'_p-\frac{1}{2}\varepsilon_{slq}\varepsilon_{kmp}\int_S \delta(\mathbf{R})\,P_{l,sm}dS'_k; \tag{3.8}$$

$$J_{kl}=\varepsilon_{pmk}\int_L \delta(\mathbf{R})\,P_l v_m dL'_p-\int_S \delta(R)\dot{P}_l dS'_k-\frac{1}{2}\int_S \delta(\mathbf{R})\,(P_{l,k}+P_{k,l})\,v_j dS'_j; \tag{3.9}$$

$$S_{kq}=\frac{1}{2}\varepsilon_{pmk}\varepsilon_{nlq}\int_L \delta(\mathbf{R})\,P_{l,n}v_m dL'_p-\frac{1}{2}\varepsilon_{nlq}\int_S \delta(\mathbf{R})\,\dot{P}_{l,n}dS'_k-\frac{1}{2}\varepsilon_{nlq}\int_S \delta(\mathbf{R})\,P_{l,nk}v_j dS'_j. \tag{3.10}$$

It is easy to see that in the case when the general expression (2.10) goes over into the particular expression (2.1), formulas (3.7)-(3.10) yield the correct formulas for the usual disclination [5].

4. The field equations to determine the elastic fields e_{mq}, $\varkappa_{kl}$, v_l, w_q whose sources are defects distributed in the body take the following form with (3.1), (3.2) and (3.3), (3.4) taken into account

$$\operatorname{rot}\begin{pmatrix}\varkappa\\ e\end{pmatrix}=\begin{pmatrix}\theta\\ \alpha\end{pmatrix}; \tag{4.1}$$

$$\frac{\partial}{\partial t}\begin{pmatrix}\varkappa\\ e\end{pmatrix}-\operatorname{grad}\begin{pmatrix}\mathbf{w}\\ \mathbf{v}\end{pmatrix}=-\begin{pmatrix}S\\ J\end{pmatrix}. \tag{4.2}$$

Equations (4.1), (4.2) must be supplemented by a dynamic equation of the theory of elasticity (we assume the bulk forces zero)

$$\sigma_{ij,j}=\rho\dot{v}_j \tag{4.3}$$

and Hooke's law

$$\sigma_{ij}=c_{ijkl}e_{kl}, \tag{4.4}$$

where ρ is the mass density and c_{ijkl} are elastic constants. Upon conservation of the compatibility conditions (3.5) and (3.6), integration of (4.1)-(4.4) yields for the elastic strains $e_{mn}(\mathbf{r},t)$ [6]

$$e_{mn}(\mathbf{r},t)=\int\{[\varepsilon_{pmk}c_{ijkl}G_{jn,i}(\mathbf{R},T)\,\alpha_{pl}(\mathbf{r}',t')-\rho\dot{G}_{ln}(\mathbf{R},T)\,J_{ml}(\mathbf{r}',t')]-\varepsilon_{pmk}[\varepsilon_{qsl}\,c_{ijkl}\,H_{jn,is}(\mathbf{R},T)\,\theta_{pq}(\mathbf{r}',t')-\rho\dot{H}_{kn,s}(\mathbf{R},T)\,S_{sp}(\mathbf{r}',t')]\}_{(mn)}\,d\mathbf{r}'dt', \tag{4.5}$$

where $G_{jn}(\mathbf{r},t)$ is the dynamical Green's function and the potential function $H_{jn}(\mathbf{r},t)$ is determined by the relationship

$$H_{jn}(\mathbf{r},t)=\int(4\pi R)^{-1}G_{jn}(\mathbf{r}',t)\,d\mathbf{r}'$$

($T=t-t'$). Hence, the elastic stresses σ_{ij} produced by the defects distributed in the body are found by substituting (4.5) into (4.4). The set of equations (4.1)-(4.4) therefore determines the dynamic state of an elastic medium for given defect characteristics.

If general Somigliani dislocations are distributed continuously in a medium, then the distribution function $f(\mathbf{r},t;q)$ can be introduced to describe them, where q is the set of generalized coordinates governing the type of Somigliani dislocation. For instance, in the case of ordinary closed dislocation loops of circular shape $q=\{r,\mathbf{n},\mathbf{b}\}$, where r is the loop radius, **n** is the vector normal to S and **b** is the Burgers dislocation vector. The mean densities $\bar{\alpha}_{pl}$, $\bar{\theta}_{pq}$ and fluxes $\bar{J}_{kl}$, $\bar{S}_{kq}$ of dislocations and disclinations over a physically small volume of a medium are here found by integration of the tensors $\alpha_{pl}(q)$, $\theta_{pq}(q)$, $J_{kl}(q)$, $S_{kq}(q)$ corresponding to an isolated general Somigliani dislocation of the type q, with respect to the generalized coordinates q. For example, we will have for $\bar{\alpha}_{pl}$

$$\bar{\alpha}_{pl} = \int \alpha_{pl}(q) f(\mathbf{r}, t; q) dq.$$

Here the distribution function f(r, t; q) is normalized so that the number of general Somigliani dislocations dN with generalized coordinates between q and q + dq per unit volume dN = f(**r**, t; q)dq. Evolution of the ensemble of defects in time can be described by using a balance equation for the distribution function f(**r**, t; q) of the form

$$\partial f/\partial t + \operatorname{div}(\mathbf{Q}f) = I(f, f'), \tag{4.6}$$

where div is the divergence operation in (q + 3)-space, Q is the velocity vector in the same space $Q = \{\dot{q}, \dot{\mathbf{r}}\}$; I(f, f') is the collision integral that takes account of the jumplike processes of the change in state of the defects (generation, combination, etc.). The dynamical law

$$\mathbf{Q} = \mathbf{Q}(\sigma_{ij}^{+}), \tag{4.7}$$

must be given to close (4.6), where σ_{ij}^{+} are the effective stresses comprised of external and internal (produced by the defects themselves) stresses. The form of the law (4.7) is determined from the solution of the problem of motion of a single defect in a field of stresses σ_{ij}^{+} (see details in [4]).

Therefore, a general model is proposed for the Somigliani dislocation that is a defect of more general type than the ordinary dislocation and disclination. The general Somigliani dislocation is defined by giving the basic plastic fields according to (2.11)-(2.14); moreover, the dislocation-disclination model (representation) with continuously distributed dislocations and disclinations (3.7)-(3.11) corresponds to it. Evolution of the ensemble of Somigliani dislocations in time can be described by a distribution function subject to a balance equation of the form (4.6).

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